Additional assignments

Week 6

Essay. Consider a game in which each player $i \in \{1, ..., n\}$ writes a real number x_i between 0 and 10 in a sealed envelope. The average $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is then computed. The winner of the game is the player whose x_i is closest to $\frac{1}{3}\bar{x}$ (one-third of the average), and receives a prize of value \$50. (In case of a tie, all the winners receive an equal share of the prize.)

What would be the value of your x_i if you were to play this game? Justify your choice, being very careful to motivate any assumption you make.

*[Hint: this game has a pure-strategy Nash equilibrium. However, **in the real world**, those who play the Nash-equilibrium strategy virtually never win. How is that? Lecture 3 on behavioral game theory may be helpful.]*

Week 8

Problem A. Consider an auction which is a variant of the sealed-bid first-price standard format. A set $N = \{1, ..., n\}$ of bidders, $n \geq 2$, bids for a single indivisible object; each bidder $i \in N$ has a private valuation v_i for the object that is distributed independently and identically over $[0, \bar{v}]$ according to a continuous, increasing cumulative distribution function F; and each bidder submits a bid, without observing the bids of other bidders. At the end of the auction, a six-faced dice is rolled. The highest bidder (any tie for the highest bid is broken with equal probability) pays her bid, but receives the object only if the dice comes up with the number 1 (i.e., with probability 1/6); if the dice comes up with any other number (i.e., with probability 5/6), the seller keeps the object *and* the highest bidder's bid. The other bidders do not pay anything.

1. Use Myerson's Lemma to determine the bidding function of each bidder *i* in a symmetric (Bayesian-Nash) equilibrium of the auction, making clear the logic of your derivation.
2. Is this auction revenue-equivalent to the standard sealed-bid first-price auction by virtue of the Revenue Equivalence Theorem? Explain briefly. Determine if the seller's expected revenue is more than (if so, by how much), less than (if so, by how much), or equal to that from a standard sealed-bid first-price auction.

Problem B. Consider two bidders bidding for a single indivisible object. It is common knowledge that each bidder i $(i = 1, 2)$ has a private valuation v_i for the object that is distributed independently and identically over $[0, 1]$ according to the uniform distribution function $F(v_i) = v_i$. By the Revenue Equivalence Theorem, the sealed-bid first-price (SBFP) and second-price (SBSP) auctions generate the same *expected* revenue for the seller. The *actual* revenue from the realized valuations of the two bidders, however, may be different. Give one example of the realizations of the valuations of the two bidders such that the SBFP auction generates more revenue than the SBSP auction and another example such that the reverse is true.

Problem C. Let $N = \{1, ..., n\}, n \geq 2$ a set of bidders with independent, private valuations drawn from a continuous uniform distribution F defined over $[0, 1]$. *[Hint: remember that the uniform distribution has the following properties:*

- $F(x) = x$
- *Let* $X \sim U[a, b]$*; then,* $\mathbb{E}(X) = \frac{a+b}{2}$
- *Let* $Z = mX + p$*; then,* $Z \sim U[ma + p, mb + p]$.
- *Let* $Y = X|X < x$ *be the random variable X except that we know that it must be inferior to a certain value* x*. (For instance, this is the distribution of the second-highest valuation in a sealed-bid second price auction, once bidder* i *finds out that he had the highest bid and that therefore all other valuations must be less than* v_i*.) Then,* Y *is still uniformly distributed:* $Y \sim U[a, x]$.

- *The expected k^{th}-highest of n values drawn from the distribution of X above is $a + \frac{n+1-k}{n+1}(b-a)$.]*

Consider the following auction formats:

1. Sealed-bid first-price
2. Sealed-bid second-price
3. Ascending
4. Descending
5. All-pay sealed bid, which is the same as the SBFP except that all bidders, not just the winner, pay their bid
6. The auction from last week's **Problem B**

In each of the above, find an expression for the following items *[in the order that you want and using any result that you want, explaining briefly why it holds]*:

- Each bidder's bidding function $\beta_i(\cdot)$ in a dominant-strategy or Bayesian Nash equilibrium
- The auctioneer's realized revenue
- The auctioneer's expected revenue
- Each bidder's expected payment
- Each bidder's expected payoff

Proof. Consider a labor market with two types of agents: job-seekers and firms. Let us assume that each firm has only one vacancy to fill, so that this becomes a one-to-one matching problem. We further require that the set J of job-seekers and the set F of firms be finite. Each job-seeker $j \in J$ has a strict preference relation defined over the firms in F and each firm $f \in F$ has a strict preference relation defined over the job-seekers in J. *[A precision on preference relations: to denote that job-seeker j_1 strictly prefers firm f_2 to f_3 and strictly prefers f_3 to f_1, we use the following notation:*

$$f_2 \succ_{j_1} f_3 \succ_{j_1} f_1$$

Note that since we require preference relations to be ***strict****, no job-seeker can be indifferent between two firms, and no firm can be indifferent between two job-seekers.]*

Let μ and μ' be two stable matchings. Prove that if a job-seeker j^* prefers her employer f^* in μ to her employer in μ', then f^* must prefer her employee in μ' to j^*. *[Think carefully about what you are trying to prove, then scrupulously analyze the information provided. In particular, you are not told how the matching is reached, so the result that you are trying to prove must hold irrespective of the matching mechanism.]*